QUEENSMEAD PRIMARY ACADEMY

Calculation Policy

Adopted: July 2020

Reviewed: September 2024
Next Review: September 2025 or when required by GAT

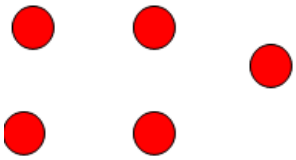
Mrs M Richards
(Assistant Principal)

HTO will replace HTU

The majority of children should be using the refined column method by the end of Year 3.

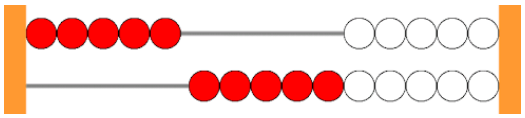
Stage 1 – Various practical activities to ensure a conceptual understanding of addition.

Children will be learning how to count objects, match to the numeral, find and write the correct number next to them.



=5

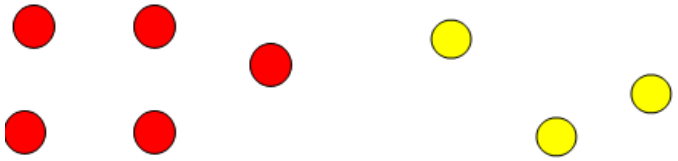
Children will also be learning how to subitise.



= 5

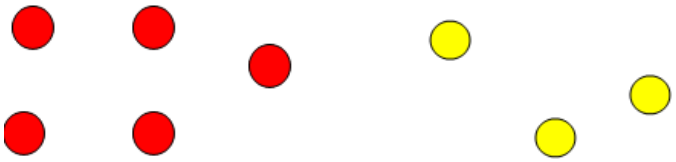
= 5

Children will then learn to combine and count two sets of objects (aggregation).



= 8

Next, children will learn to add on to an existing set (augmentation). This means that they will start from 5 and count 3 more to 8.

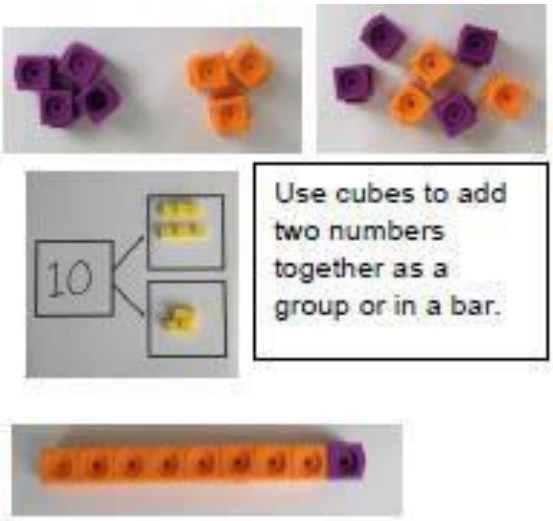


5 + 3 = 8

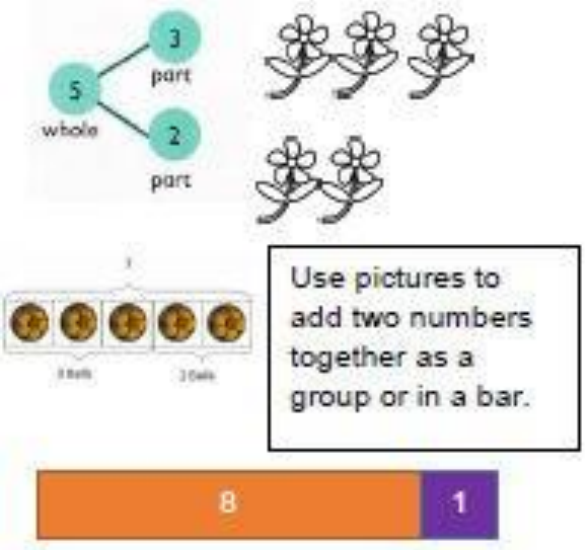
Augmentation is the first step towards a written calculation and children will be using the '+' and '=' symbols.

Combining two parts to make a whole: part-part whole model

Concrete



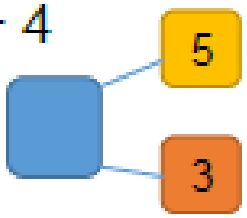
Pictorial



Abstract

$4 + 3 = 7$

$10 = 6 + 4$



Use the part-part whole diagram as shown above to move into the abstract.

Stage 2a – Using a numberline alongside the concrete objects

The majority of children should be at Stage 2a by the end of Year 1.

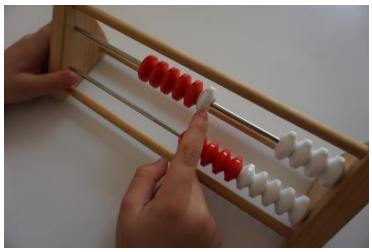
Objectives from NC2014 (Year 1):

- Read, write and interpret mathematical statements involving addition and equals signs.
- Represent and use number bonds within 20.
- Add one-digit and two-digit numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9.

Starting at the bigger number and counting on in ones

Concrete

Rekenreks



Please refer to the NCETM Mastering Number resources for the many ways that this resource can be used to support understanding.

Start with the larger number on the rekenrek (using one push).

Numicon

Please refer to the Numicon resources for the many ways that this resource can be used to support understanding.

6 + 5

+

Bead Strings

Start with the larger number on the bead string and then count on to the smaller number one by one to find the answer.

12 + 5 = 17

Pictorial

Using number tracks:

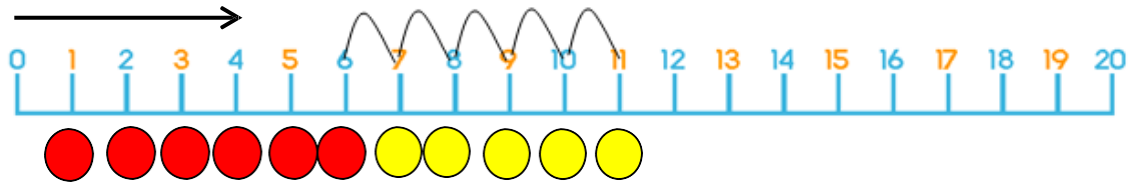
Start at the larger number on the number track and count on in ones to find the answer.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Using a number line:

Start at the larger number on the number line and count on in ones to find the answer.

6 + 5 = 11



Use concrete objects alongside a number line to move from concrete to pictorial.

As children become more confident with numbers, they will begin to use an empty number line. They will be using base 10 materials or place value counters.

Abstract

Place the larger number in your head and count on the smaller number to find the answer.

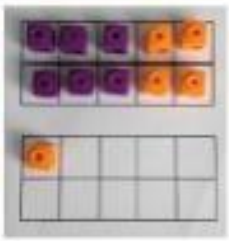
5 + 12 = 17

Regrouping to make 10

Children are encouraged to use their knowledge of number bonds to help them partition numbers.

Concrete

Start with the bigger number and use the smaller number to make 10:



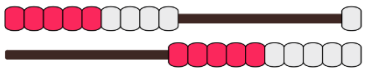
6 + 5

6 + 4 + 1 = 11

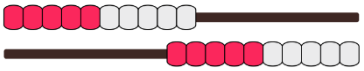
9 + 3

9 + 1 + 2 = 12

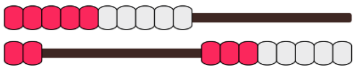
9 + 3. Make 9



Add 1 to make 10

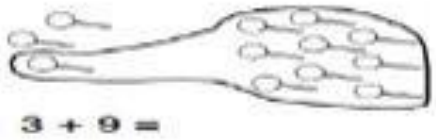


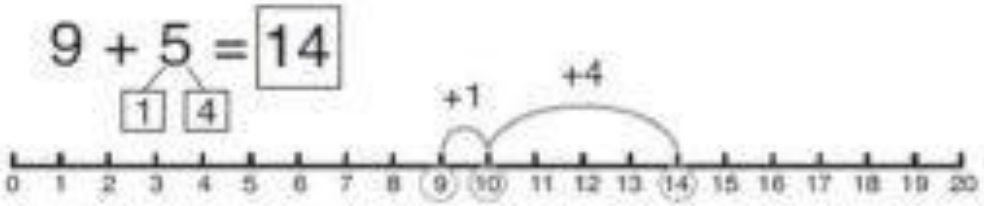
Then add 2. 9 + 1 + 2 = 12



Pictorial

Use pictures or a number line. Regroup or partition the smaller number to make 10.





Abstract

If I am at seven, how many more do I need to make 10?

How many more do I add on now?

7 + 4 = 11

Stage 2b – Using a numberline alongside the concrete objects (larger jumps)

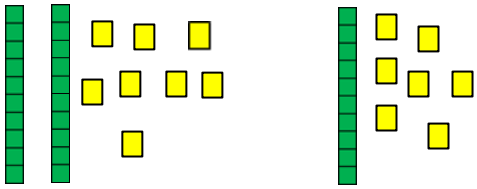
The majority of children should be at Stage 2b during Year 2, progressing to Stage 3 when ready.

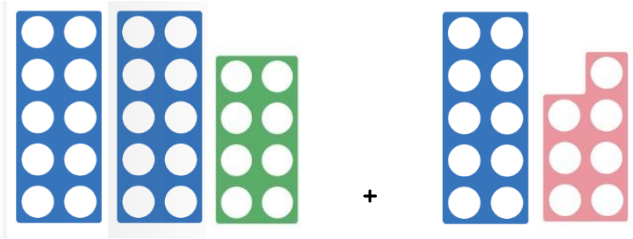
Objectives from NC2014 (Year 2):

- Solve problems with addition:
  - Using concrete objects and pictorial representations, including those involving numbers, quantities and measures.
  - Applying their increasing knowledge of mental and written methods.
- Recall and use addition facts to 20 fluently, and derive and use related facts up to 100.
- Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers and three one-digit numbers.
- Show that addition of two numbers can be done in any order (commutative).
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

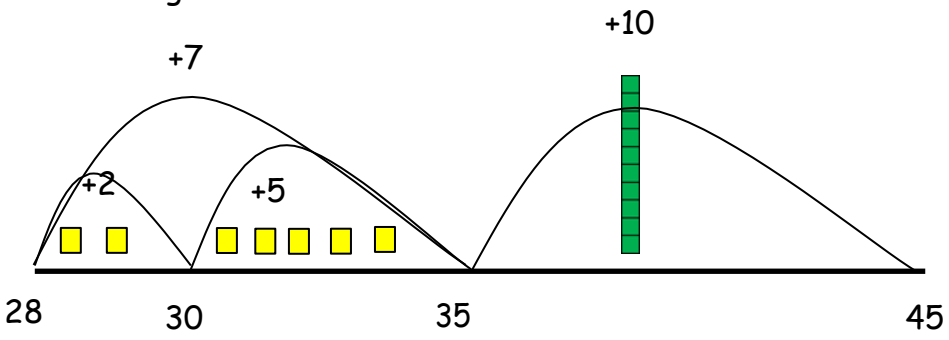
They can now add in bigger jumps.

28 + 17





Adding the ones first



Add the ones then the 10

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Remember:

- **Children also need to opportunities to add more than 2 numbers together (see next page).**
- **Ensure that estimation is encouraged and then checked using the inverse operation.**
- **Introducing the children to the inverse operation concept is essential. They should be shown that 12 + 5 = 17 and 5 + 12 = 17 so 17 – 5 = 12 and 17 – 12 = 5**

Adding three single digits

Concrete

Make 10 with two of the numbers (if possible) then add on the third number:

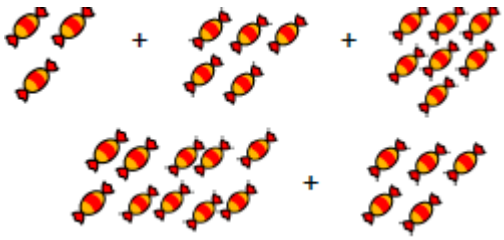
4 + 7 + 6 = 17

Put 6 and 4 together to make 10. Add on 7.

Pictorial

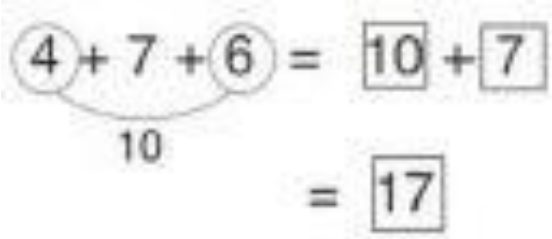
Add together three groups of objects.

Draw a picture to recombine the groups to make 10.



Abstract

Combine the two numbers that make 10 and then add on the remainder.



Stage 3 – Expanded Column Method – using partitioning

The majority of children should be at Stage 3 by the end of Year 2.

The children now need to move to a more refined method. The expanded column method prepares the children well for stage 4, the refined column method (abstract).

- **Ensure that estimation and checking using the inverse operation are a part of everyday practise.**
- **Adding more than 2 two-digit numbers is also expected at this stage.**

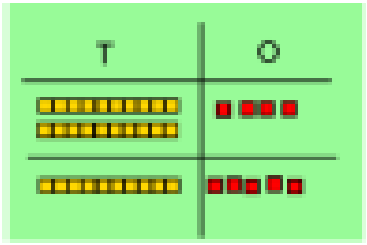
Once again, concrete objects can be used alongside this method to support understanding (see below).

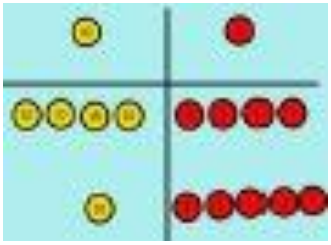
Concrete

Start with no regrouping.

Add together the ones first then add the tens.

Use the base 10 blocks first before moving onto place value counters.



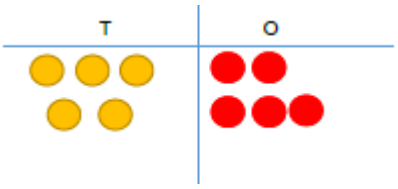


24 + 15

Pictorial

Start with no regrouping.

After practically using the base 10 blocks and place value counters, children can draw the counters to help them solve additions:



Concrete

Move on to regrouping:

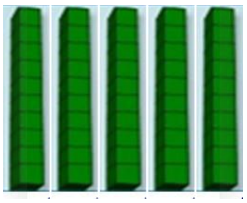
The place value counters and base 10 materials help the children progress towards a more familiar written algorithm.

59 + 22 =

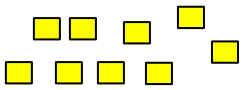
50	**9**
20	**2**
70 +	**11 = 81**

59 + 22

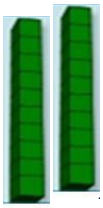
Tens Ones



50

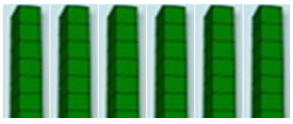


9

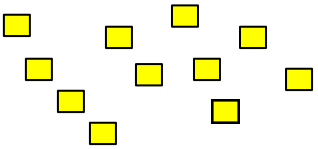


20

2

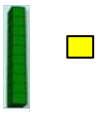


70



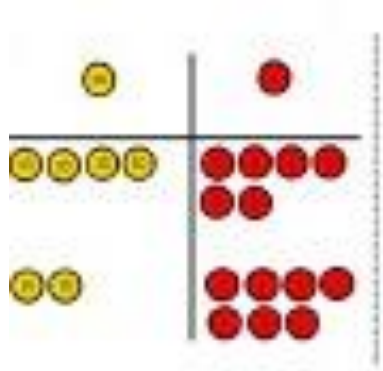
11

OR



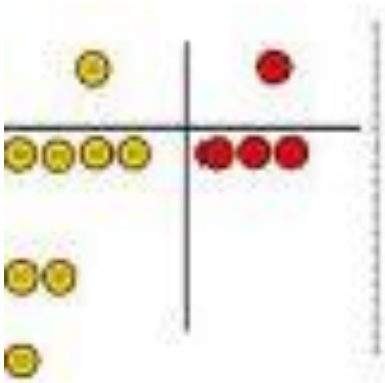
= 81

Make both numbers on a place value grid:



$$\begin{array}{r} 46 \\ +\ 27 \\ \hline \end{array}$$

Add up the ones and exchange 10 ones for 1 ten:



$$\begin{array}{r} 46 \\ +\ 27 \\ \hline \end{array}$$

Add up the rest of the columns, exchanging when there are 10 counters in one column for the next place value column until every column has been added.

Pictorial

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.

Abstract

Move onto two exchanges when ready.

TO + TO

87 + 49

80	**7**
40	**9**
120	**16**

120 + 16 = 136

then money (use coins alongside this method initially)

83p + 78p

80p	**3p**
70p	**8p**
150p	**11p**

150p + 11p = 161p
or
£1.50 £1.61

Stage 4 – The Refined Column Method

The majority of children should be at Stage 4 by the end of Year 3.

Objectives from NC2014 (Year 3):

- Add numbers mentally, including: a three-digit number and ones; a three-digit number and tens and a three-digit number and hundreds.
- Add numbers with up to three digits, using the formal written method of columnar addition.
- Estimate the answer to a calculation and use inverse operations to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

This method needs to be introduced alongside the expanded method (stage 3) to ensure that the children make the link between the two methods.

Avoid phrases, such as, 'carrying'. Explain to the children that the 'ten number goes in the tens' column to be added with the tens' or 'the hundred number goes in the hundreds' column to be added with the hundreds'.

Remember to use concrete and pictorial representations until children are secure.

Abstract

67 + 49

Estimate:

70 + 50 = 120

	H	T	O
		6	7
+		4	9
	1	1	
	1	1	6

The place value columns can be omitted when secure.

The 'ten' goes in the tens' column to be added with the other tens.

The numbers will sit on the line (not under). Remind children to leave a space before the 'equals' line when setting out addition.

Inverse:

116 – 49 = 67

Stage 4 – The Refined Column Method: Examples linked to NC2014 Year 4, 5 & 6 objectives

The majority of children should be at Stage 4 by the end of Year 3.

Objectives from NC2014 (Year 4):

- Add numbers with up to 4 digits using the formal written method of columnar addition where appropriate.
- Estimate and use inverse operations to check answers to a calculation.
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Remember to use concrete and pictorial representations until children are secure.

Abstract

1765 + 4388

Estimate:

1800 + 4400 = 6200

```
  1765
+ 4388
  111
  6153
```

Inverse:

6153 – 4388 = 1765

Objectives from NC2014 (Year 5):

- Add whole numbers with more than 4 digits, including using the formal written method.
- Add numbers mentally with increasingly large numbers.
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Remember to use concrete and pictorial representations until children are secure.

As children move on to decimals, money and decimal place value counters can be used to support.

Abstract

89787 + 6879

Estimate:

90000 + 7000 = 97000

```
  89787
+  6879
  1111
  96666
```

Inverse:

96666 – 6879 = 89787

Objectives from NC2014 (Year 6):

- Perform mental calculations, including with mixed operations and large numbers.
- Use their knowledge of the order of operations to carry out calculations involving the four operations.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Remember to use concrete and pictorial representations until children are secure.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Abstract

£87.76 + £348.68

Estimate:
90 + 350 = 440

£348.68
+ £ 87.76
1 1 1 1
£436.44

Inverse:
£436.44 – £87.76 = £348.68

52.85 + 143.6

Estimate:
53 + 144 = 197

52.85
+ 143.6
1
196.45

Inverse:
196.45 – 143.6 = 52.85

Subtraction

Stage 1 – Various practical activities to ensure a conceptual understanding of subtraction.

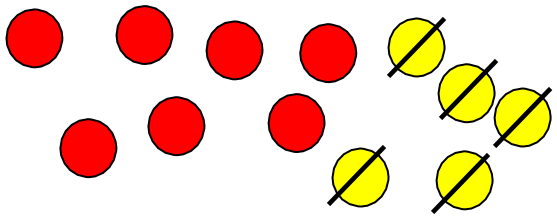
Taking away ones:

Concrete

Children will be learning how to subtract using different objects or toys to support them:

Pictorial

Cross out drawn objects to show what has been taken away:



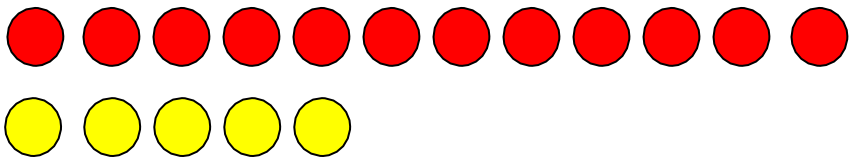
Abstract

The notation will come later after practical activities & language is secure:

12 – 5 = 7

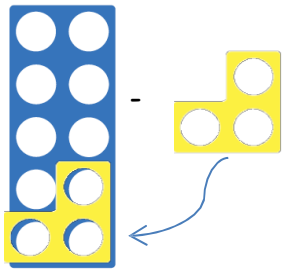
Comparing a set of objects (comparison or difference):

This helps children understanding the inverse and commutative nature of mathematics.



It is expected that Numicon is used to support the understanding of subtraction.

Please refer to the Numicon resources for the many ways that this resource can be used to support the understanding.



Lay the Numicon on top to show 10-3 = 7
This introduces 'finding the difference'

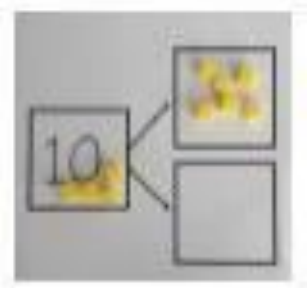
Part-Part Whole Model for subtraction:

Concrete

Link to addition – use the part-part whole model to help explain the inverse relationship between addition and subtraction:

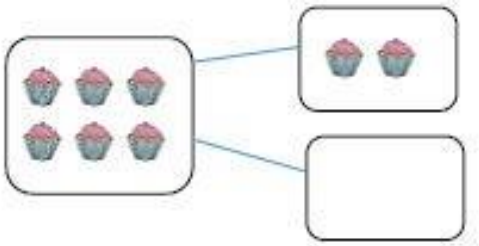
10 – 6 =

If 10 is the whole and 6 is one of the parts, what is the other part?



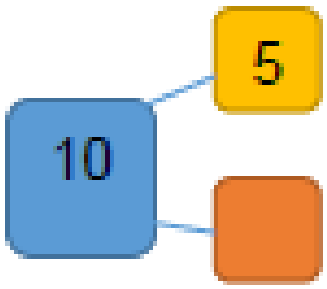
Pictorial

Use a pictorial representation of objects to show the part-part whole model:



Abstract

Move to using numbers within the part-part whole model:



Stage 2a – Counting back

The majority of children should be at Stage 2a by the end of Year 1.

Objectives from NC2014 (Year 1):

- Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs.
- Represent and use number bonds and related subtraction facts within 20.
- Subtract one-digit and two-digit numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9.

Counting back in ones:

Concrete

Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones:

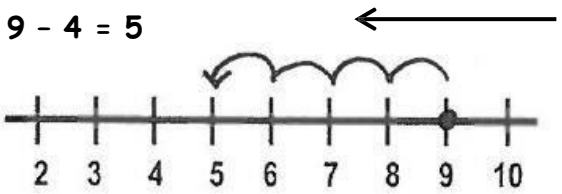
13 – 4

Use counters and move them away from the group as you take them away, counting backwards as you do:

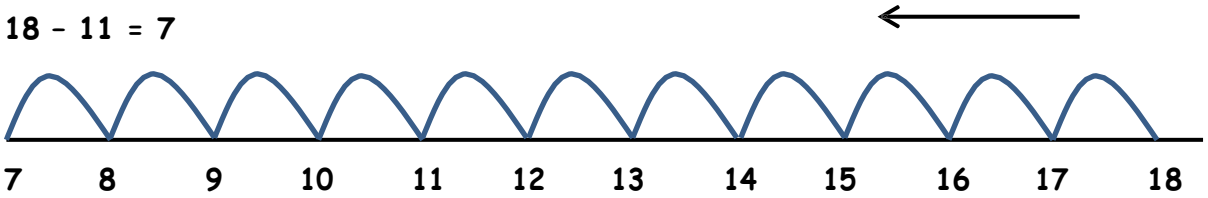
Pictorial

Count back on a number line or number track:

9 – 4 = 5



18 – 11 = 7



Start at the bigger number and count back the smaller number showing the jumps on the number line.

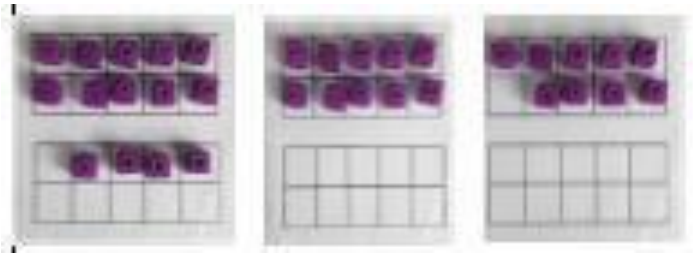
This method would initially be modelled to the children alongside the concrete objects.

Make 10:

Concrete

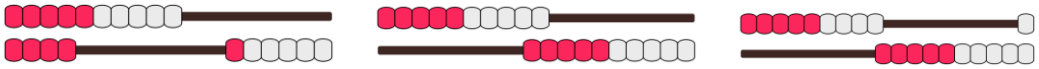
14 – 5 =

Tens Frames

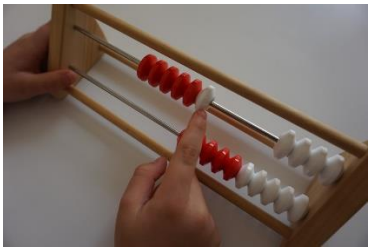


Make 14. Take away 4. Take away 1 more.
You are left with the answer of 9.

Rekenreks



Make 14. Take away 4. Take away 1 more.
You are left with the answer of 9.

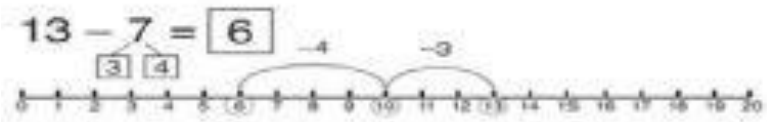


Please refer to the NCETM Mastering Number resources for the many ways that this resource can be used to support understanding.

Pictorial

13 – 7 =

Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer of 6:



Abstract

16 – 8 =

How many do we need to take off to reach the next 10?

How many do we have left to take off?

Stage 2b – Counting back (2-digit numbers > 20)

The majority of children should be at Stage 2b during Year 2, progressing to Stage 3 when ready.

Objectives from NC2014 (Year 2):

- Solve problems with subtraction:
  - Using concrete objects and pictorial representations, including those involving numbers, quantities and measures.
  - Applying their increasing knowledge of mental and written methods.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

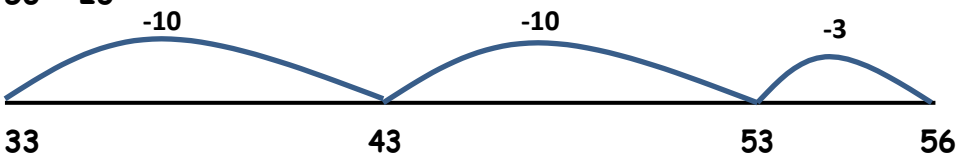
Concrete - see Stage 2a

Use concrete objects (e.g. bead strings) alongside the pictorial representations until children are secure.

Pictorial

Count back on a number line:

56 – 23 = **Subtract the 'ones' first**



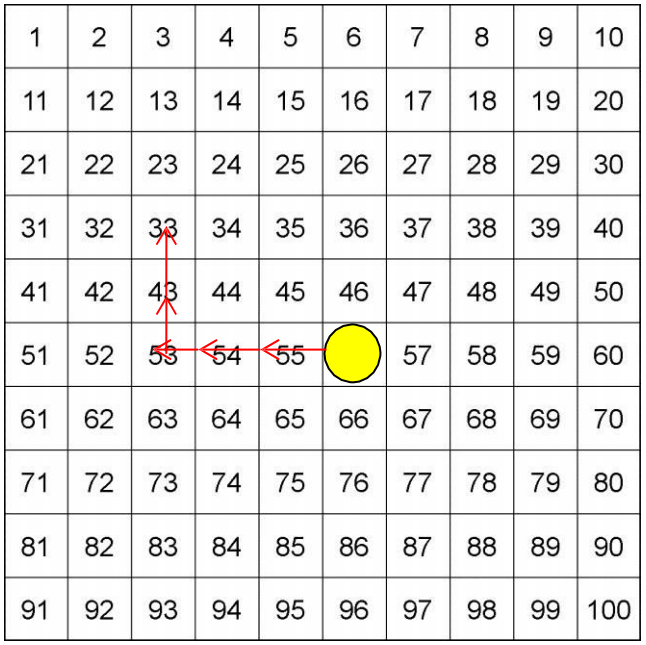
Start at the bigger number and count back the smaller number showing the jumps on the number line. Subtract the ones first, then the tens.

This method would initially be modelled to the children alongside the concrete objects.

Counting back can also be modelled using a number square (grid). Children should know that moving to the left on a grid is -1 and moving up is -10.

56 – 23 = 33

Subtract the ones first



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Abstract

Put 13 in your head, count back 4.

What number are you at?

Use your fingers to help.

Introducing the children to the inverse operation concept is essential and should be modelled together. This should be shown as

12 + 5 = 17 and 5 + 12 = 17 so 17 – 5 = 12 and 17 – 12 = 5

Stage 3a – Counting on – finding the difference.

The majority of children should be at Stage 3a by the end of Year 2.

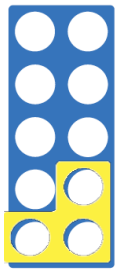
Once the children are secure with what subtraction is (counting back) and can show this using concrete objects, move the children onto the 'counting on' method – finding the difference.

Concrete

Compare amounts and objects to find the difference:

Revisit using the Numicon to secure the conceptual understanding:

10 – 3 =



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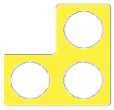
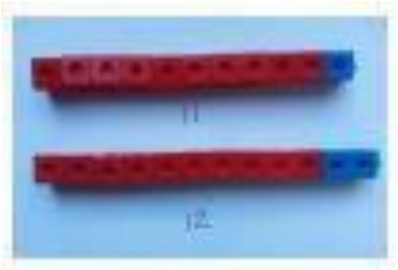


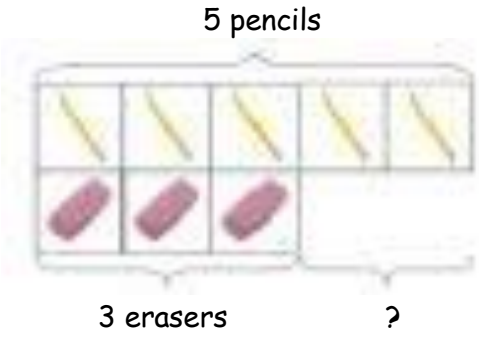
Encourage 'counting on' from 3 to 10

So 3 + 7 more equals 10.

Use cubes to build towers or make bars to find the difference:



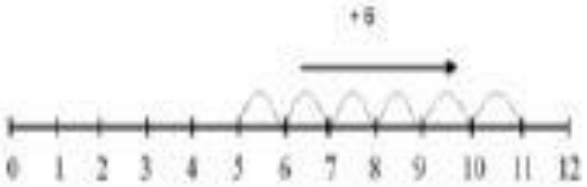
Use basic bar models with items to find the difference:



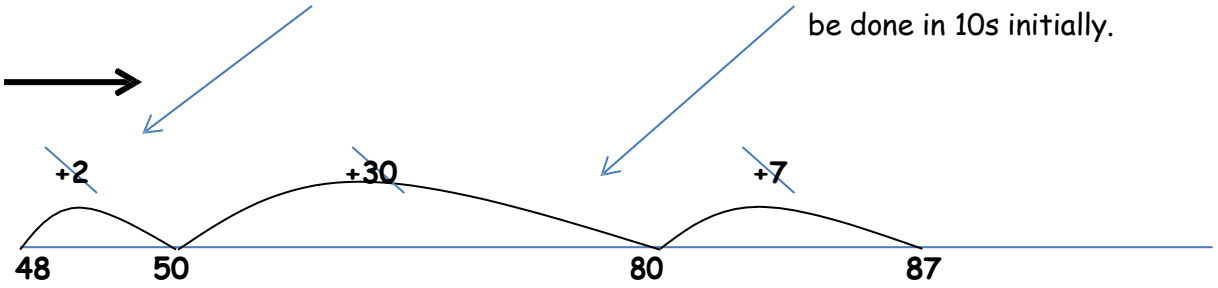
Pictorial

Numberlines

Count on the numberline to find the difference:



87 – 48 = Always go to the next 10 The 10 below the number – this can be done in 10s initially.



Re-combining

30 + 7 + 2

37 + 2 = 39

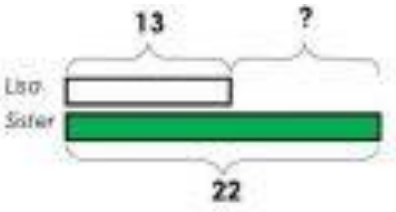
Encourage the children to use the inverse to check their answer e.g. 39 + 48 = 87

The 'jumps' need to be added up below the numberline. Encourage the children to add the largest number first and cross them out as they go along to help to avoid mistakes.

Comparison bar models

Draw bars to find the difference between 2 numbers:

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



Abstract

Hannah has 23 sandwiches. Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Stage 3b – Counting on using a numberline – finding the difference (numbers > 100).

The majority of children should be at Stage 3b during Year 3, progressing to Stage 4 when ready.

Objectives from NC2014 (Year 3):

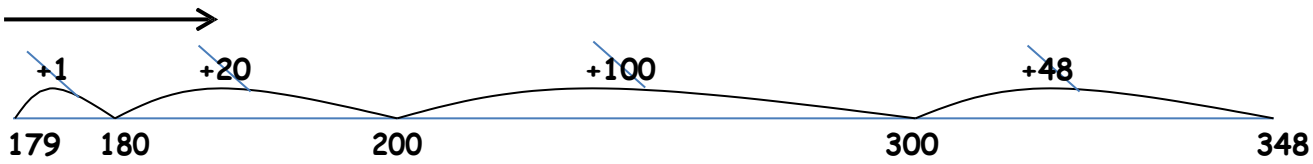
- Subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens and a three-digit number and hundreds.
- Subtract numbers with up to three digits, using the formal written method of columnar subtraction.
- Estimate the answer to a calculation and use inverse operations to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Provide opportunities to subtract HTO – TO (e.g. 243 – 86)

When ready for larger numbers…

Pictorial

348 - 179



100 + 48 + 20 + 1

148 + 21 = 169

This is an example of a jotting to support a mental calculation so can be shown in different ways.

Stage 4 – The Expanded Column Method – using partitioning

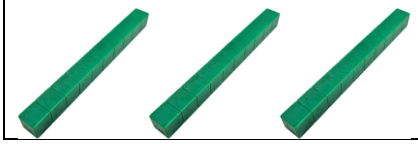
The majority of children should be at Stage 4 by the end of Year 3.

Start without regrouping:

Concrete

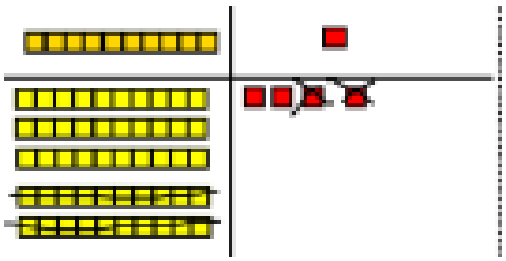
Use base 10 to make the larger number then take the smaller number away.

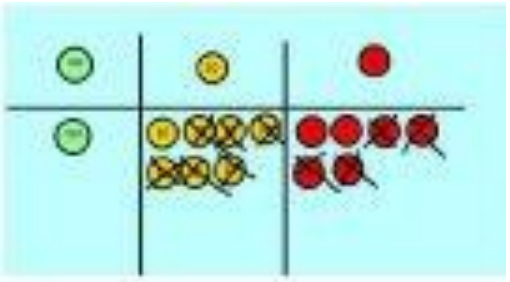
e.g. 53 – 21

Use base 10 to make the larger number, 53 (5 tens and 3 ones):	
	
Take away the smaller number, 21 (subtract ones first):	
	

Pictorial

Draw the base 10 or place value counters on a place value grid and show what you have taken away by crossing the counters out:

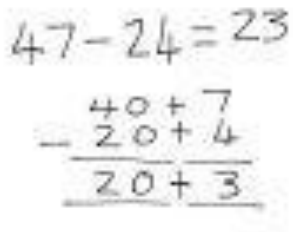




Abstract

Initially draw the base 10 or place value counters alongside the written calculation to help show working.

This should lead to a clear written expanded column subtraction:



Move on to regrouping:

Concrete

This method should initially be modelled alongside the Base 10, place value counters, Numicon or other concrete resources. Physically show the children the exchanging process.

Use base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with two exchanges.

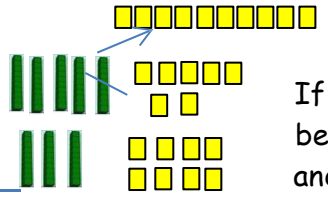
Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Estimate:
60 - 40 = 20

The language and explanation used when modelling this method is important and needs to be consistent.

57 – 38 =

40	1
~~50~~	7
30	8
10	9



Use this explanation:

If you have 7 'ones' you do not have enough 'ones' to be able to subtract (take away) 8 – (in this context and without going into negative numbers) - so a ten needs to be **exchanged**.

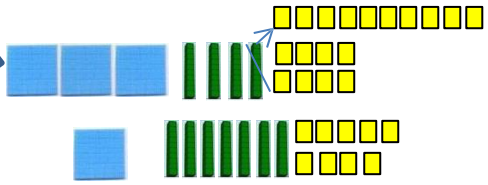
Inverse:
19 + 38 = 57

Estimate:

350 - 180 = 170

348 – 179

200	1 30	1
~~300~~	~~40~~	8
100	70	9
100	60	9



If you have 8 'ones' you do not have enough 'ones' to be able to subtract (take away) 9 – (in this context and without going into negative numbers) - so a ten needs to be **exchanged**.

Please do not use other phrases, such as, 'borrowing', 'knocking on the door' etc.

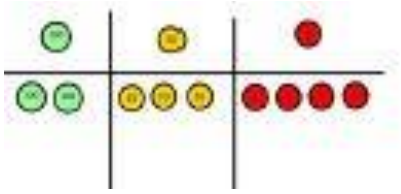
Inverse:

169 + 179 = 348

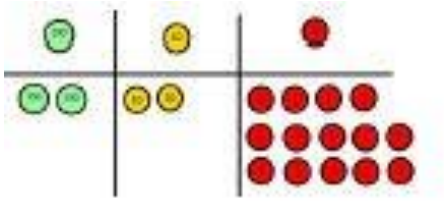
<u>Using place value counters</u>

234 – 88

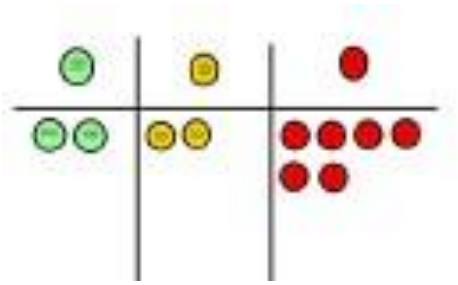
Make the larger number with the place value counters:



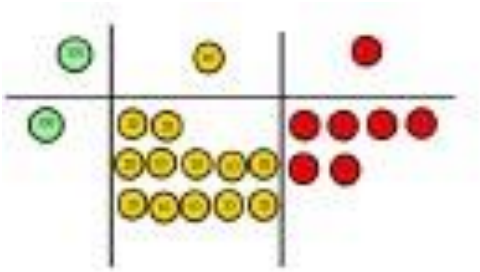
Start with the ones. Can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones:



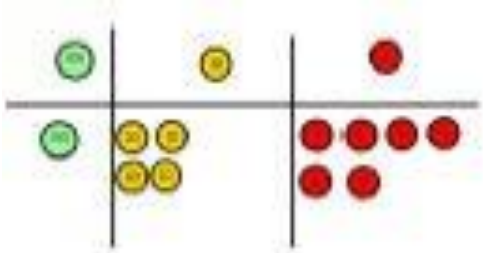
Now I can subtract my ones:



Now look at the tens. Can I take away 8 tens easily? I need to exchange 1 hundred for 10 tens:

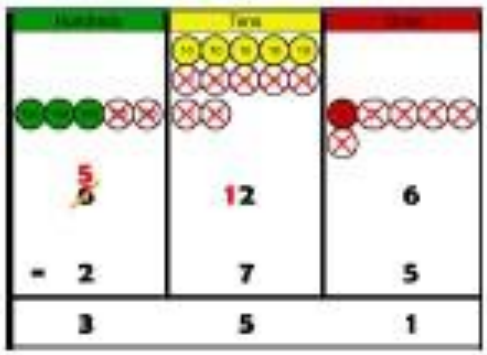


Now I can take away 8 tens and complete my subtraction:



Pictorial

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing all the exchanges you make:



Abstract

Children can start their formal written method by partitioning the number into clear place value columns:

Stage 5 – The Refined Column Method

The majority of children should be at Stage 5 by the end of Year 4.

Objectives from NC2014 (Year 4):

- Subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate.
- Estimate and use inverse operations to check answers to a calculation.
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Remember to use concrete and pictorial representations until children are secure.

Abstract

348 – 179 =

Estimate:

350 - 180 = 170

$$\begin{array}{r} {\scriptstyle 2\ \ 13\ \ 1} \\ 3\ \not{4}\ \not{8} \\ -\ 1\ 7\ 9 \\ \hline 1\ 6\ 9 \end{array}$$

Inverse:

169 + 179 = 348

The term 'exchange' must be used throughout this method.

'If you have 8 ones you do not have enough ones to be able to subtract (take away) 9 – (in this context and without going into negative numbers) - so a ten needs to be **exchanged for 10 ones**.

Please do not use other phrases, such as, 'borrowing', 'knocking on the door' etc.

3952 – 1475 =

Estimate:

4000 - 1500 = 2500

$$\begin{array}{r} {\scriptstyle \ \ 8\ \ 14\ \ 1} \\ 3\ \ \not{9}\ \ \not{5}\ \ 2 \\ -\ 1\ \ 4\ \ 7\ \ 5 \\ \hline 2\ \ 4\ \ 7\ \ 7 \end{array}$$

Inverse:

2477 + 1475 = 3952

Stage 5 – The Refined Column Method: Examples linked to NC2014 Year 5 & 6 objectives

The majority of children should be at Stage 5 by the end of Year 4.

Objectives from NC2014 (Year 5):

- Subtract whole numbers with more than 4 digits, including using the formal written method.
- Subtract numbers mentally with increasingly large numbers.
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Remember to use concrete and pictorial representations until children are secure.

Abstract

19.076 – 3.142 =

Estimate:

19 – 3 = 16

$$\begin{array}{r} {\scriptstyle 8\quad 1} \\ 1\,\not{9}\,.\,0\,7\,6 \\ -\quad 3\,.\,1\,4\,2 \\ \hline 1\,5\,.\,9\,3\,4 \end{array}$$

Inverse:

15.934 + 3.142 = 19.076

Objectives from NC2014 (Year 6):

- Perform mental calculations, including with mixed operations and large numbers.
- Use their knowledge of the order of operations to carry out calculations involving the four operations.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Remember to use concrete and pictorial representations until children are secure.

Abstract

8.6045 – 3.758 =

Estimate:

9 – 4 = 5

$$\begin{array}{r} {\scriptstyle 7\quad 15\quad 1\,\not{9}\quad 1} \\ 8\,.\,\not{6}\quad\not{0}\quad 4\quad 5 \\ -\ 3\,.\,7\quad 5\quad 8 \\ \hline 4\,.\,8\quad 4\quad 6\quad 5 \end{array}$$

Inverse:

4.8465 + 3.758 = 8.6045

Multiplication

The majority of children should be using the formal written methods of short and long multiplication by the end of Year 5.

4 x3 should be explained as 4 three times (4+4+4). The starting number is the number you start with and the x? tells you what to do with it. So, the calculation becomes 4, multiplied by 3.

Stage 1

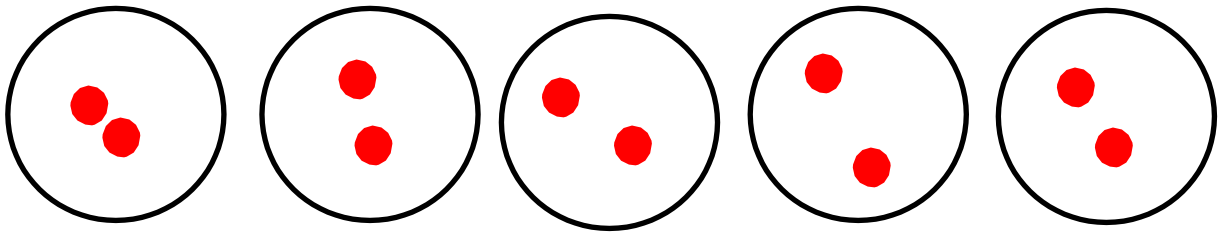
The majority of children should be at Stage 1 by the end of Year 1.

Objectives from NC2014 (Year 1):

- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with support of the teacher.

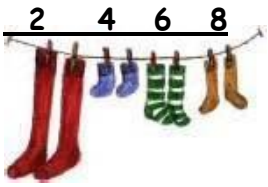
Counting in 2s, 5s, 10s using practical resources – Concrete

2 x5



2 4 6 8 10

Use socks, cubes and a range of other resources to model the concept of multiplication.



2 4 6 8

2 + 2 + 2 + 2 **2 x4**

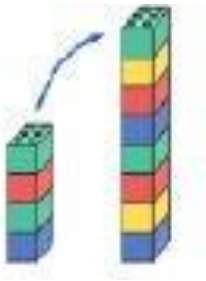
3 x4 **3** **6** **9** **12**

Use the language '4 groups of 3' and '4 lots of 3' with the children but emphasise 'groups of' as this will then support the next stage.

Doubling

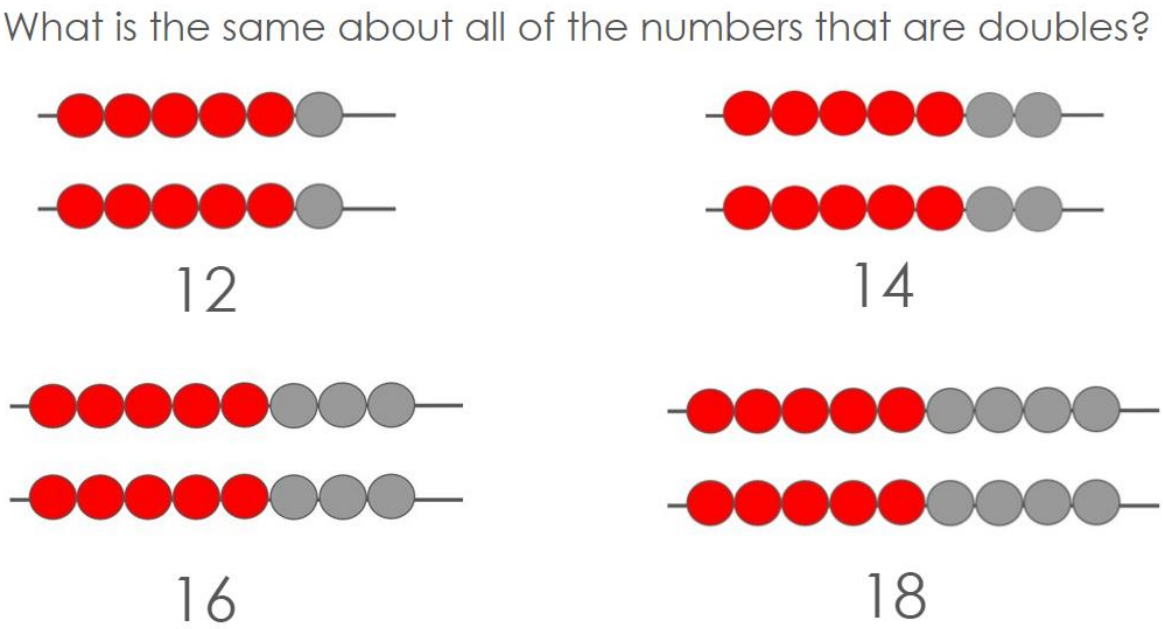
Concrete

Use practical activities to show how to double a number:



Double 4 is 8 4 x 2 = 8

Use rekenreks to explore doubles:



Taken from: Mastering Number Y2 materials 2021/22 ncetm.org.uk

Pictorial

Draw pictures to show how to double a number:

Double 4 is 8 4 x 2 = 8

Stage 2

The majority of children should be at Stage 2 by the end of Year 2.

Objectives from NC2014 (Year 2):

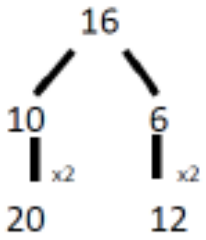
- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs.
- Show that multiplication of two numbers can be done in any order (commutative).
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts.

Doubling

NOTE: Ensure children are secure with Concrete and Pictorial stages of doubling before moving on to abstract (see Multiplication Stage 1).

Abstract

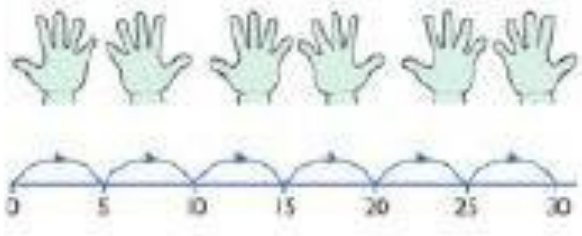
Partition a number and then double each part before recombining it back together:



Counting in multiples

NOTE: Ensure children are secure with Concrete stage of counting in multiples before moving on to the pictorial and abstract (see Multiplication Stage 1).

Pictorial



Abstract

Count in multiples of a number aloud.

Write sequences with multiples of numbers.

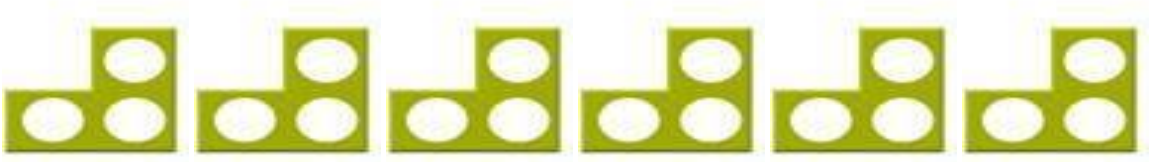
e.g. 2, 4, 6, 8, 10.

Repeated Addition

Concrete

Use the Numicon to model this concept.

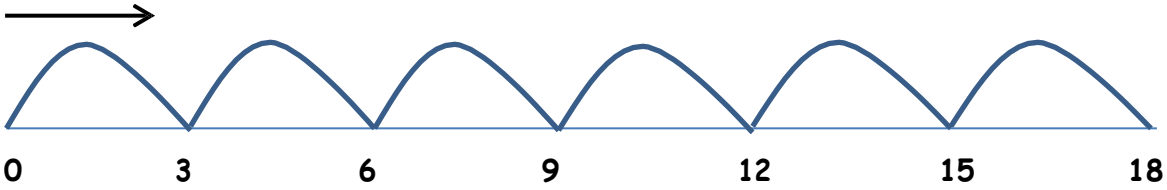
3 x6



3 + 3 + 3 + 3 + 3 + 3

Pictorial

This can then be linked to a numberline. Show these images together.



Also, use counters alongside the numberline to show this in various ways.

Abstract

Write addition sentences to describe objects and pictures:

Arrays

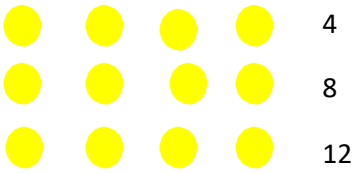
Children will become familiar with using arrays to model x tables.

Concrete

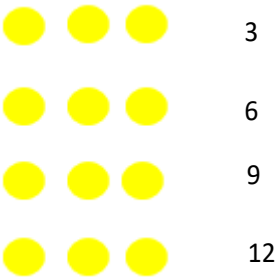
Create arrays using counters / cubes to show multiplication sentences:

Pictorial

Draw arrays in different rotations to find commutative multiplication sentences:

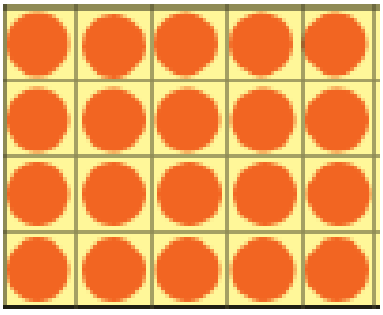


4 x3



3 x4

Link arrays to area of rectangles:



Abstract

Use an array to write multiplication sentences and reinforce repeated addition:

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

5 x 3 = 15

3 x 5 = 15

Stage 3 – Partitioning

The majority of children should be at Stage 3 by the end of Year 3.

Objectives from NC2014 (Year 3):

- Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.
- Write and calculate mathematical statements for multiplication using the multiplication tables they know, including for two-digit times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

When practising written methods, avoid multiplying by 2, 4 or 5. The children need to consider more effective mental methods for these calculations (e.g. x4 is double and double again).

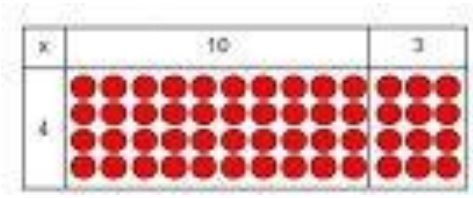
Multiplying by a single digit

23 x6 =
3 x6 = 18
20 x6 = 120

120 + 18 = 138

Grid Method

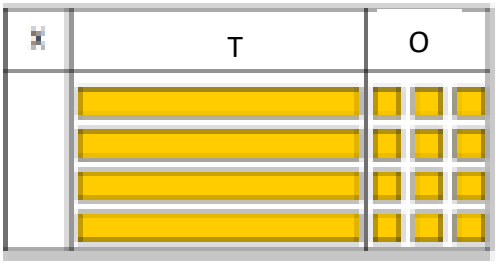
Concrete
Show the link with arrays first to introduce the grid method:



4 rows of 10 (A row of 10, 4 times)

4 rows of 3 (A row of 3, 4 times)

Move on to using Base 10 to move towards a more compact method:

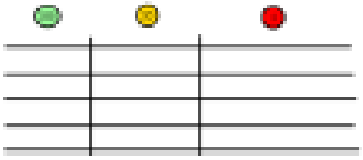


4 rows of 13 (A row of 13, 4 times)

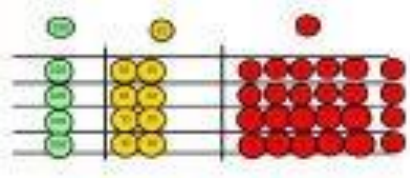
Move on to place value counters to show how we are finding groups of a number:

126 x 4 =

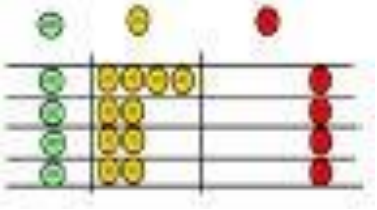
We are multiplying by 4 so we need 4 rows:



Fill each row with 126:



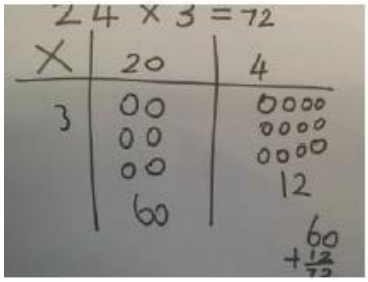
Starting with the ones, add up each column. Make any exchanges needed:



Then you have your answer.

Pictorial

Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking:



Abstract

Start with multiplying the one-digit numbers.

x	30	5
7	30x7 3x7 = 21 210	7x5 7x5 = 35 35

When the grid is complete, add up the answers outside of the grid:

210 + 35

The jottings inside the grid can be dropped when secure.

Stage 4 – The Formal Written Method of Short Multiplication

The majority of children should be at Stage 4 by the end of Year 4.

Stage 4 will only be used when the children are showing a very secure understanding of stage 3.

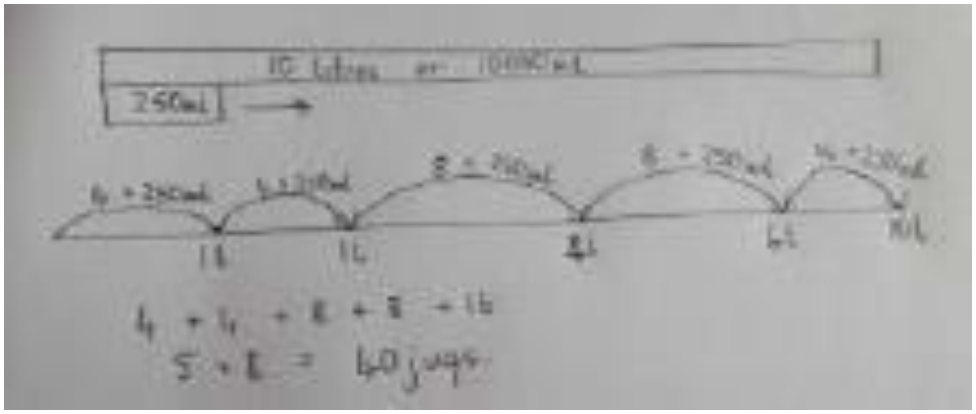
Objectives from NC2014 (Year 4):

- Recall multiplication facts for multiplication tables up to 12 x 12.
- Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers.
- Recognise and use factor pairs and commutativity in mental calculations.
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Stage 4

Concrete & Pictorial – see Stage 3 examples.

Bar modelling and number lines can support children when solving problems with multiplication **alongside** the formal written methods:



Abstract

Stage 4a (The expanded method of short multiplication)

Use the expanded method initially:

H	T	O		
	2	4		
x		6		
	2	4	(4 x6)	
1	2	0	(20 x6)	
1	4	4		

Show the grid method alongside

x	20	4
6	120	24

120 + 24 = 144

Stage 4b (The compact method of short multiplication)

Move on to the compact method when children are secure with stage 4a.

NOTE: Show this method alongside the expanded method (above) initially.

H	T	O
	2	4
x		6
	2	
1	4	4

Th	H	T	O
	3	4	2
x			7
	2	1	
2	3	9	4

Stage 5 – The Formal Written Method of Long Multiplication

The majority of children should be at Stage 5 by the end of Year 5.

Objectives from NC2014 (Year 5):

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.
- Multiply numbers mentally drawing upon known facts.
- Multiply whole numbers and those involving decimals by 10, 100 and 1000.

Multiplying by a two-digit number

NOTE: The grid method (stage 3) can also be used when multiplying a two-digit by a two-digit number.

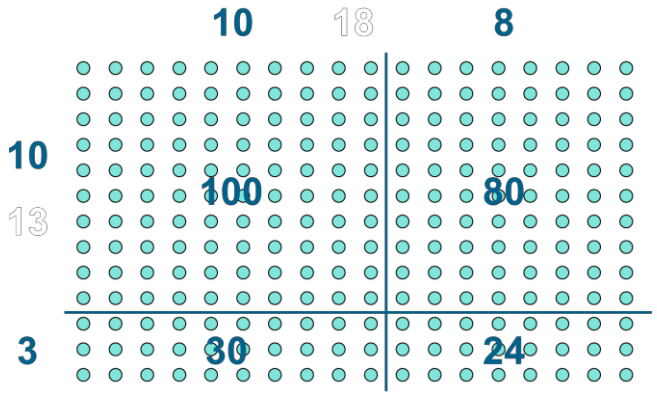
Concrete & Pictorial

Start by reminding the children of what multiplication is before moving on to the

formal written method of long multiplication.

18 x13 =

X	10	8	
10			
3			



100 + 80 + 30 + 24 (The children may wish to show this in a column method format.)

Abstract

Moving on to Stage 5a (The expanded method of long multiplication)

Use the expanded method initially:

H	T	O	
	3	5	
x	2	6	
	3	0	(5 x6)
1	8	0	(30 x6)
1	0	0	(5 x20)
6	0	0	(30 x20)
1			
9	1	0	

Show the grid method alongside

x	30	5
20	20x30 2x3 = 6 600	20x5 2x5 = 10 100
6	30x6 3x6 = 18 180	6x5 30

Stage 5b (The compact method of long multiplication)

Move on to the compact method when children are secure with stage 5a.

NOTE: Show this method alongside the expanded method (above) initially.

	H	T	O
		3	5
x		2	6
		3	
	2	1	0
	1		
+	7	0	0
	9	1	0

Show this initially…

35 x6 = 210

35 x20 = 700

Th	H	T	O
	1	2	4
x		2	6
	1	2	
	7	4	4
+			
2	4	8	0
1	1		
3	2	2	4

Cross out multiplication exchanges to avoid confusion when adding products.

Stage 5 – The Formal Written Method of Long Multiplication: Examples linked to the NC2014 Year 6 objectives

The majority of children should be at Stage 5 by the end of Year 5.

Objectives from NC2014 (Year 6):

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Use their knowledge of the order of operations to carry out calculations involving the four operations.
- Solve problems involving addition, subtraction, multiplication and division.
- Use estimation to check answers to calculations and to determine, in the context of a problem, an appropriate degree of accuracy.

Abstract

Estimate:

1431 x 20 = 28620

		1	4	3	1
x				2	3
		1			
+		4	2	9	3	(1431 x3)
	2	8	6	2	0	(1431 x20)
	1		1		
	3	2	9	1	3

4.65 x 26 =

Cross out multiplication exchanges to avoid confusion when adding products.

Estimate:

5 x 26 = 130

			4	.	6	5	
x			2		6		
			3		3		
+		2	7	.	9	0	(4.65 x6)
		1	1				
		9	3	.	0	0	(4.65 x20)
		1					
	1	2	0	.	9	0	

Division

The majority of children should be using the formal written methods of short and long division by the end of Year 6.

Stage 1 – Introducing the concept of sharing through the use of a range of practical resources and discussion. Language is important.

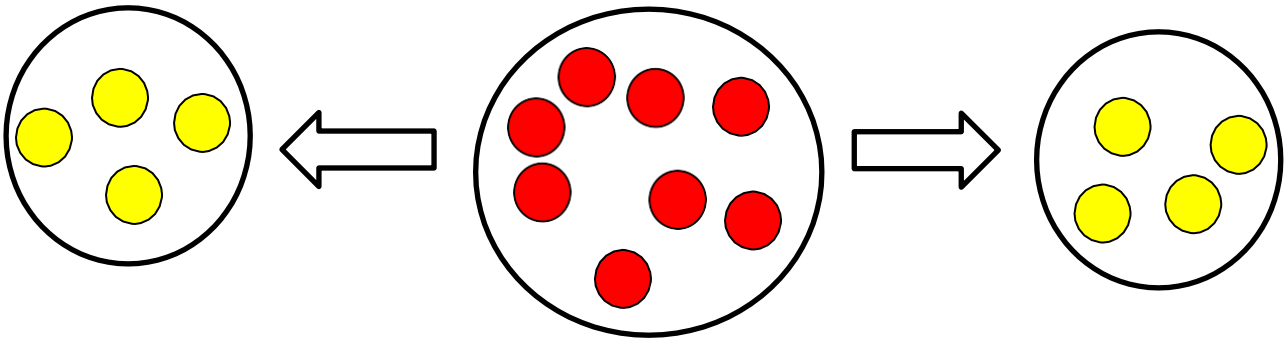
The majority of children should be at Stage 1 during Year 1, progressing to Stage 2 when ready.

Objectives from NC2014 (Year 1):

- Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with support of the teacher.

Concrete

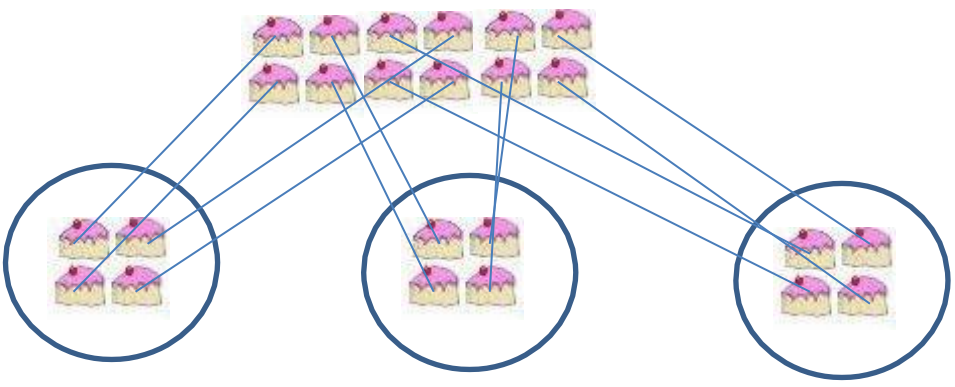
Children will be learning how to share, using different objects or toys to support them.



Share between 2, then 3, etc.

This experience will be brought into school from children's own experiences.

Share 12 cakes shared between 3 people (12÷3=)



Pictorial

Children use pictures or shapes to share quantities:

Share 8 flowers between 2 people (8 ÷ 2 =):

Abstract

Share 9 buns between 3 people:

9 ÷ 3 = 3

Halving

Concrete

Children will be learning to find half of a whole.

Halve the cake:

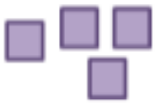
Children will also be learning to halve an equal quantity.
Use practical activities to show how to find half of a number:

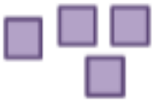
Half of 8 is 4

8 ÷ 2 = 4

Pictorial

Draw pictures to show how to find half of a number:





Half of 8 is 4

8 ÷ 2 = 4

Abstract

Half of 8

8 ÷ 2 = 4

Stage 2 - Grouping

The majority of children should be at Stage 2 by the end of Year 1.

Objectives from NC2014 (Year 1):

- Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with support of the teacher.

Notes and guidance (non-statutory):

Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.

They make connections between arrays, number patterns, and counting in twos, fives and tens.

Sharing becomes inefficient as children learn bigger numbers. Grouping should then be modelled.

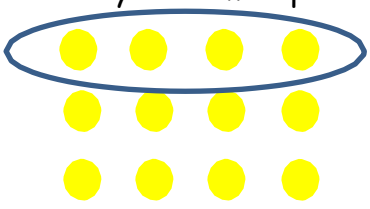
Concrete

Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding:

10 ÷ 2 =

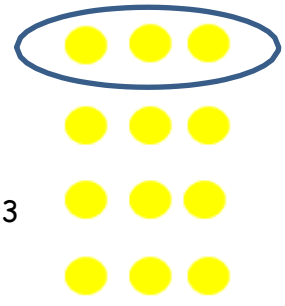
12 ÷ 4 =

Link to arrays and multiplication:



1 group

3 groups of 4
4 x3



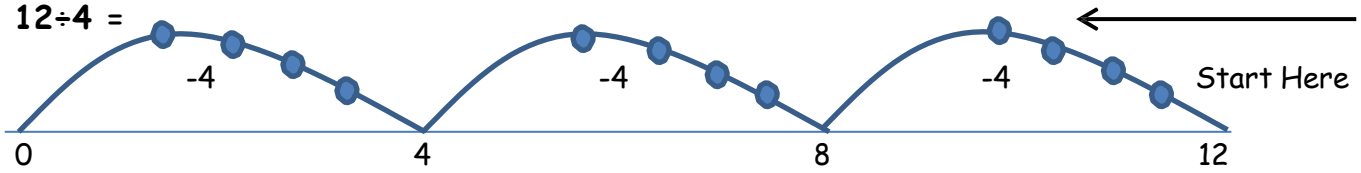
4 groups of 3
3 x4
12÷3

Pictorial

Number line:

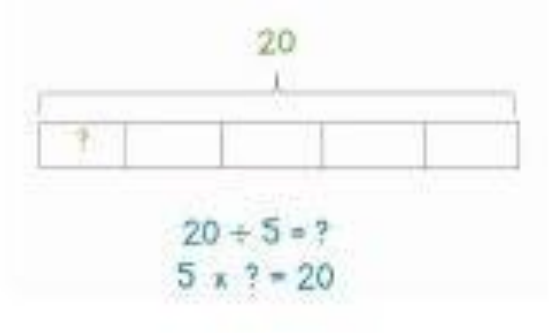
Use a number line to show jumps in groups. The number of jumps equals the number of groups:

12÷4 =



Bar Model:

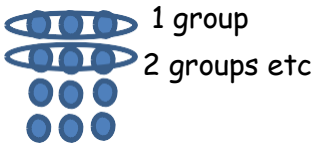
Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group:

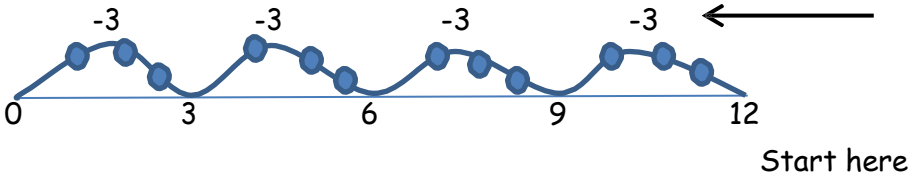


Ensure that the children have a clear understanding of what division is – make the link between sharing and grouping e.g.

'Division is when objects are shared. Sharing is not an effective method when dealing with larger numbers so another approach is to group the objects. Division is subtracting groups of objects'.

12÷3 = starting with 12 objects and then subtracting (taking away) groups of 3 to find out how many groups of 3 there are in 12. This is repeated subtraction.





Abstract

Divide 28 into 7 groups. How many are in each group?

28 ÷ 7 = 4

Stage 3a – Counting back on a numberline – repeated subtraction

The majority of children should be at Stage 3a by the end of Year 2.

Objectives from NC2014 (Year 2):

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.
- Show that division of one number by another cannot be done in any order.
- Solve problems involving division, using materials, arrays, mental methods, and multiplication facts, including problems in contexts.

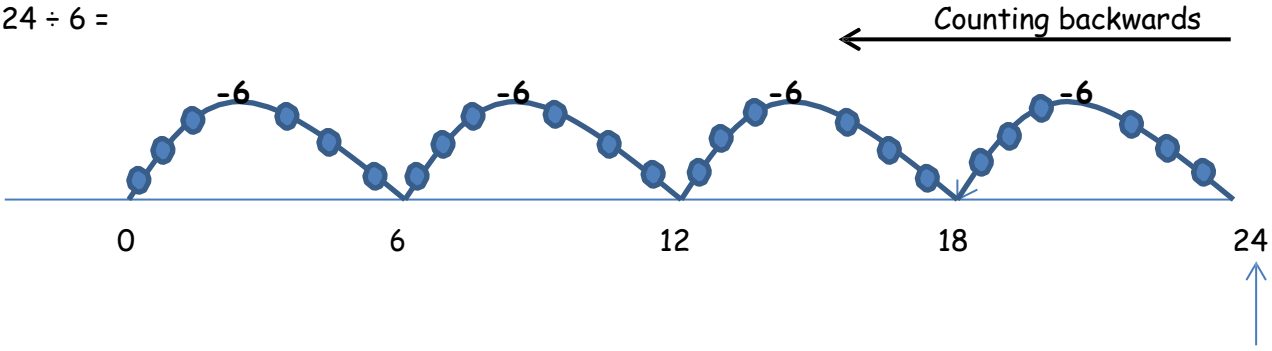
When practising written methods, avoid dividing by 2, 4 or 5. The children need to consider more effective mental methods for these calculations (e.g. ÷4 is half and half again).

Pictorial (alongside concrete)

Refer back to Stage 2 as needed to make the link between grouping and repeated subtraction.

Model this method alongside practical equipment (concrete).

24 ÷ 6 =



Show the children 24 objects. Subtract groups of 6 and place along the numberline. **The answer is 4 groups of 6 equal 24 because 6 x4 =24.**

Using Numicon to support understanding:

One way it can be used – lay out the number being divided e.g. 24 ÷ 6

Place as many of the '6' Numicon shapes on top as will fit.

This is also good to show the concept on 'remainders'.

Provide opportunities for the children to experience remainders using this method, e.g. 32÷7= 4r 4. Get the children to use the inverse operation to check the answer (7x4=28 + 4).

Stage 3b – Counting back on a numberline – repeated subtraction (multiple groups)

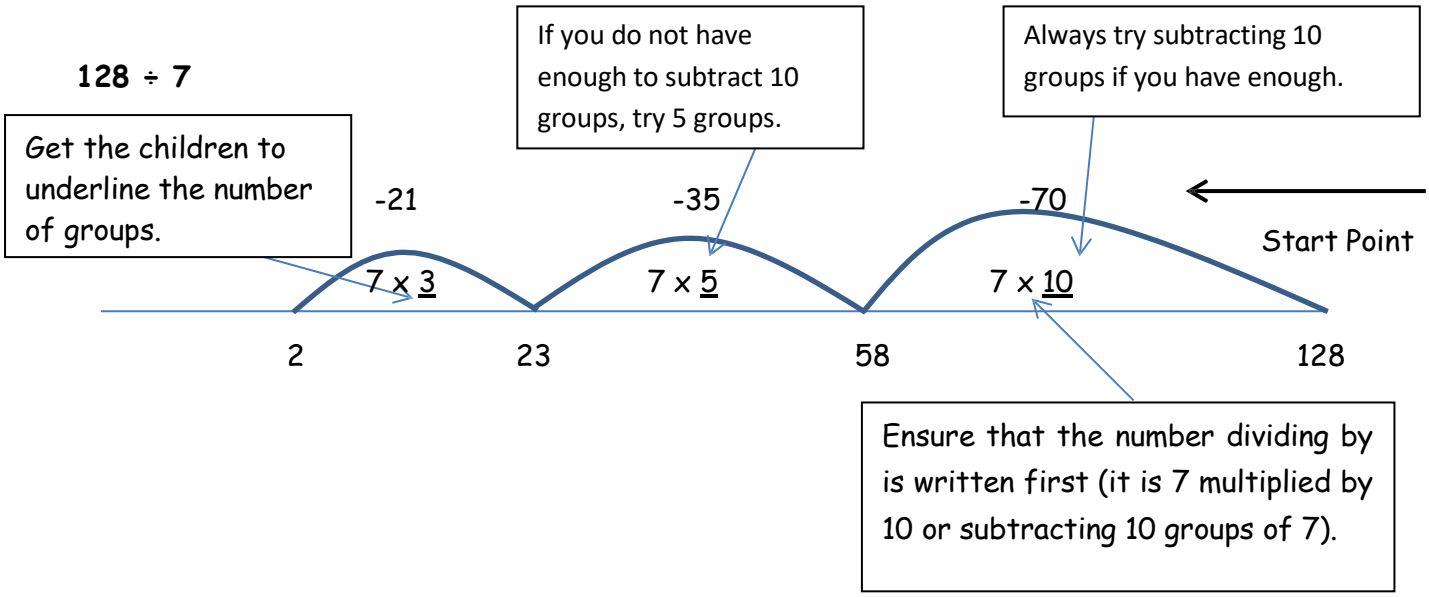
The majority of children should be at Stage 3b by the end of Year 3.

Objectives from NC2014 (Year 3):

- Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.
- Write and calculate mathematical statements for division using the multiplication tables they know, including for two-digit times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Pictorial

When secure, show the children how to subtract multiple groups. Explain how this strategy is required when the numbers are larger to ensure greater efficiency.



After completing the subtraction of the groups, count the number of groups – if these are underlined it makes it clearer.

Stage 4 – The Formal Written Method of Short Division

The majority of children should be at Stage 4 by the end of Year 4.

Objectives from NC2014 (Year 4):

- Recall multiplication and division facts for multiplication tables up to 12 x 12.
- Use place value, known and derived facts to divide mentally, including dividing by 1.
- Recognise and use factor pairs and commutativity in mental calculations.

Notes and guidance (non-statutory):

Pupils practise to become fluent in the formal written method of short division with exact answers.

Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers.

Grouping methods rely on use of multiplication facts to derive new facts. Sometimes the chunking method can be inefficient and can lead to unnecessary errors.

The formal written method of short division emphasises the positional side of place value. Therefore, to ensure conceptual understanding, this method should initially be modelled with a simple calculation alongside base ten equipment, such as place value counters (see next page for an example).

Concrete

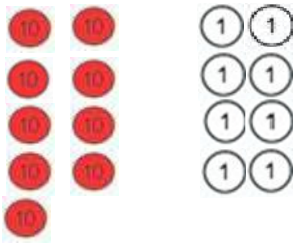
For example:

98 ÷ 7 =

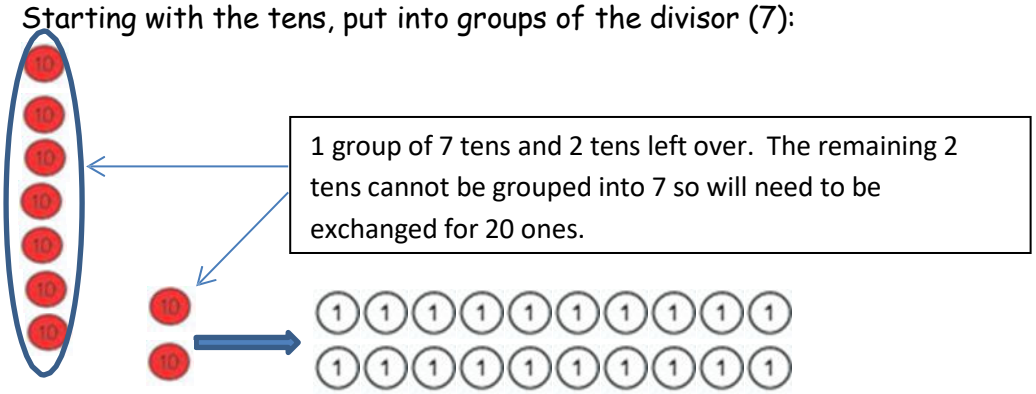
Estimate:

105 ÷ 7 = 15

Use place value counters to make the dividend (98):

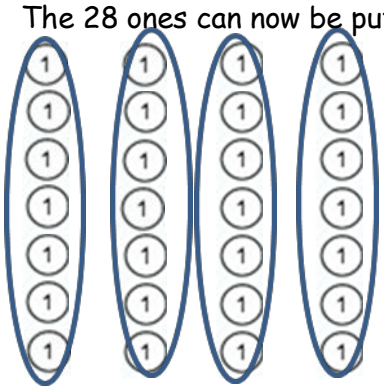


Starting with the tens, put into groups of the divisor (7):



1 group of 7 tens and 2 tens left over. The remaining 2 tens cannot be grouped into 7 so will need to be exchanged for 20 ones.

The 28 ones can now be put into groups of the divisor (7):



4 groups of 7 ones (no remainders)

1 group of 7 tens with remainder

Inverse:

14 x 7 = 98

The written calculation looks like this:

```
   1 4
  ---
7 | 9 8
    2
```

4 group of 7 ones (no remainder)

Encourage pupils to move on to pictorial representations (drawing place value counters) when ready.

Stage 5 – The Formal Written Method of Short Division: Examples linked to the NC2014 Year 5 objectives

The majority of children should be at Stage 5 by the end of Year 4.

Objectives from NC2014 (Year 5):

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Multiply numbers mentally drawing upon known facts.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Abstract - Remember, use concrete and pictorial representations until children are secure.

196 ÷ 6 =

Estimate:

180 ÷ 6 = 30

```
    0 3 2  r 4
   -----------
  |   1
6 | 1 9 6
```

Inverse:

32 x 6 + 4 = 196

432 ÷ 5 =

Estimate:

400 ÷ 5 = 80

NOTE: Remainders can also be expressed as a fraction or decimal. For example: remainder 2, 2/5 or 0.4

```
    0 8 6  r 2
   -----------
  |   3
5 | 4 3 2
```

Inverse:

86 x 5 + 2 = 432

37.2 ÷ 8 =

Estimate:

40 ÷ 8 = 5

NOTE: When dividing decimal numbers, continue to put a 0 after the least significant digit as needed (without changing the value of the dividend) until there is no remainder.

```
    0 4 . 6 5
   -----------
  |     5  4
8 | 3 7 . 2 0
```

Inverse:

4.65 x 8 = 37.2

Stage 6 – Dividing by two-digit numbers (Effective use of Short Division with multiples of the divisor).

The majority of children should be at Stage 6 by the end of Year 6.

Objectives from NC2014 (Year 6):

- Divide numbers up to 4 digits by a two-digit number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Use their knowledge of the order of operations to carry out calculations involving the four operations.
- Solve problems involving addition, subtraction, multiplication and division.
- Use estimation to check answers to calculations and to determine, in the context of a problem, an appropriate degree of accuracy.

Pupils should consider the two-digit divisor and decide whether to record multiples of the divisor.

Short Division

496 ÷ 11 =

NOTE: In this example, the divisor is still small (although a two-digit number) and the multiples of this number are known (11 times tables). So this calculation can easily be carried out using short division.

Estimate:

500 ÷ 10 = 50

```
    045  r 1
    ---
    5
11 | 496
```

OR $45\frac{1}{11}$

Inverse:

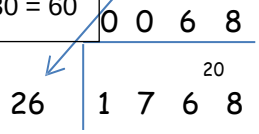
45 x 11 + 1 = 496

1,768 ÷ 26 =

NOTE: In this example, write down the multiples of the divisor.

Estimate:

1,800 ÷ 30 = 60



Inverse:

68 x 26 = 1,768

Multiples of the divisor:

26

52

78

104

130

156

182

208